

Quadratic Inequalities $y \geq ax^2 + bx + c$

$$y \geq x^2 - 2x - 15$$

1. Change the inequality symbol to an equal sign until we are ready to draw the curve.

$$y = 1x^2 - 2x - 15$$

2. Identify the **coefficients** $a=1$, $b=-2$, $c=-15$,
 - $a=1$ is called the **leading coefficient**.

3. First solve by the **Quadratic Formula**,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

The **discriminant** is used to determine the number of solutions to the QE.

- $b^2 - 4ac < 0$ is negative No Solutions
- $b^2 - 4ac = 0$ is negative One Solutions
- $b^2 - 4ac > 0$ is positive Two Solutions

- Substitute the coefficients into the discriminant $b^2 - 4ac$. $(-2)^2 - 4(1)(-15)$
- $b^2 - 4ac = +64$ The discriminant is positive so we will have two **roots** (or **zeros**).
- Substitute the coefficients into the Quadratic Formula:
- $x = \frac{-(-2) \pm \sqrt{(64)}}{2(1)} = \frac{2 \pm 8}{2}$ therefore

$$x = \frac{2+8}{2} = \frac{10}{2} = 5 \text{ and } x = \frac{2-8}{2} = \frac{-6}{2} = -3$$
- Roots are $(5, 0)$ and $(-3, 0)$

4. Second, solve by **factoring** the equation: $y = x^2 - 2x - 15$

- Can we factor by Guess & Check? **Yes** {if we could not we could *complete the square*}

$$y = x^2 - 2x - 15$$

$$y = (x-5)(x+3)$$

- Set $y=0$ $y = (x-5)(x+3) = 0$
- Use the **Zero Products Rule**: $(x-5)(x+3) = 0$
- $x-5=0$ and $x+3=0$
- Solve the Equations for the roots: $x=5$ and $x=-3$
- Write the roots as zero points: $(5, 0)$ and $(-3, 0)$

5. *Compare the results* of the Quadratic Formula and Setting $y=0$.

6. Now **Complete the Square** $x^2 + bx + \left(\frac{b}{2}\right)^2$ to find the **vertex of the parabola**.

- $y = x^2 - 2x - 15$ $b = -2$, $\frac{b}{2} = \frac{-2}{2} = -1$, $(-1)^2 = 1$

- Add 1 and subtract 1, use parathensis () around the perfect square

$$y = (x^2 - 2x + 1) - 15 - 1$$

- Factor the perfect square and simplify the constants. $y = (x - 1)^2 - 16$

7. This is the **Vertex form of the Quadratic Equation** is $y = (x - h)^2 + k$ (h, k) .

- $y = (x - 1)^2 - 16$ the vertex point is $(1, -16)$.

8. Compare the table values of the two equation forms:

x	$y = x^2 - 2x - 15$	$y = (x - 1)^2 - 16$	(x, y)	Details
-10	105	105	$(-10, 105)$	
-5	20	20	$(-5, 20)$	
-3	0	0	$(-3, 0)$	Root
0	-15	-15	$(0, -15)$	
1	-16	-16	$(1, -16)$	Vertex (Minimum)
5	0	0	$(5, 0)$	Root
10	65	65	$(10, 65)$	

9. Compare the graphs of the two equations forms using the TI-83:

10. Plot the points on Graph Paper & Label the Points, Zeros and Vertex.

11. Quadratic Inequality is $y \geq x^2 - 2x - 15$

- The symbol \geq means greater than or equal to
- Shade about the curve and make the line solid.

12. Use the TI-83 and graph the inequality.

13. Choose a y value and draw the one dimensional inequality.

Summarize what it all means. (reflection)